Lesson 5: Exponents

Student Outcomes

- Students discover that $3x = x + x + x$ is not the same thing as x^3 which is $x \cdot x \cdot x$.
- Students understand that a base number can be represented with a positive whole number, positive fraction, or positive decimal and that for any number a, we define a^m to be the product of m factors of a. The number a is the base and m is called the exponent or power of a.

Classwork

~~4×4×4×4×4×4~~

~~10×10×10×10~~

Opening Exercise

As you evaluate these expressions, pay attention to how you arrived at your answers.

$4+4+4+4+4+4+4+4+4+4+4$ 4×10=40 = 10×4=40

4, 8, 12, 16

$9+9+9+9+9$ 5×9=45

$10+10+10+10+10$ 5×10=50

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Examples 1–5

1. $5 \times 5 \times 5 \times 5 \times 5 =$ $5^5 = 3{,}125$

2. $2 \times 2 \times 2 \times 2 =$ $2^4 = 16$

Series of Products

3. $8^3 =$ $8 \times 8 \times 8 = 512$

4. $10^6 =$ Exponent

Base

$10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1{,}000{,}000$

5. $g^3 =$ $g \times g \times g$ / $g \cdot g \cdot g$

Go back to Examples 1 – 4 and use a calculator to evaluate the expressions.

What is the difference between $3g$ and g^3?

$3g = g + g + g$

$g^3 = g \times g \times g$

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Examples 6–8

Series of Products

6. $3.8^4 =$ $3.8 \times 3.8 \times 3.8 \times 3.8$

7. $2.1 \times 2.1 =$ $2.1^2 = 2\frac{1}{10} \times 2\frac{1}{10} = \left(2\frac{1}{10}\right)^2$

8. $0.75 \times 0.75 \times 0.75 =$ $0.75^3 = \frac{75}{100} \times \frac{75}{100} \times \frac{75}{100} = \left(\frac{75}{100}\right)^3$

The base number can also be a fraction. Convert the decimals to fractions in Examples 7 and 8 and evaluate. Leave your answer as a fraction. Remember how to multiply fractions!

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Examples 9–10

9. $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

10. $\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

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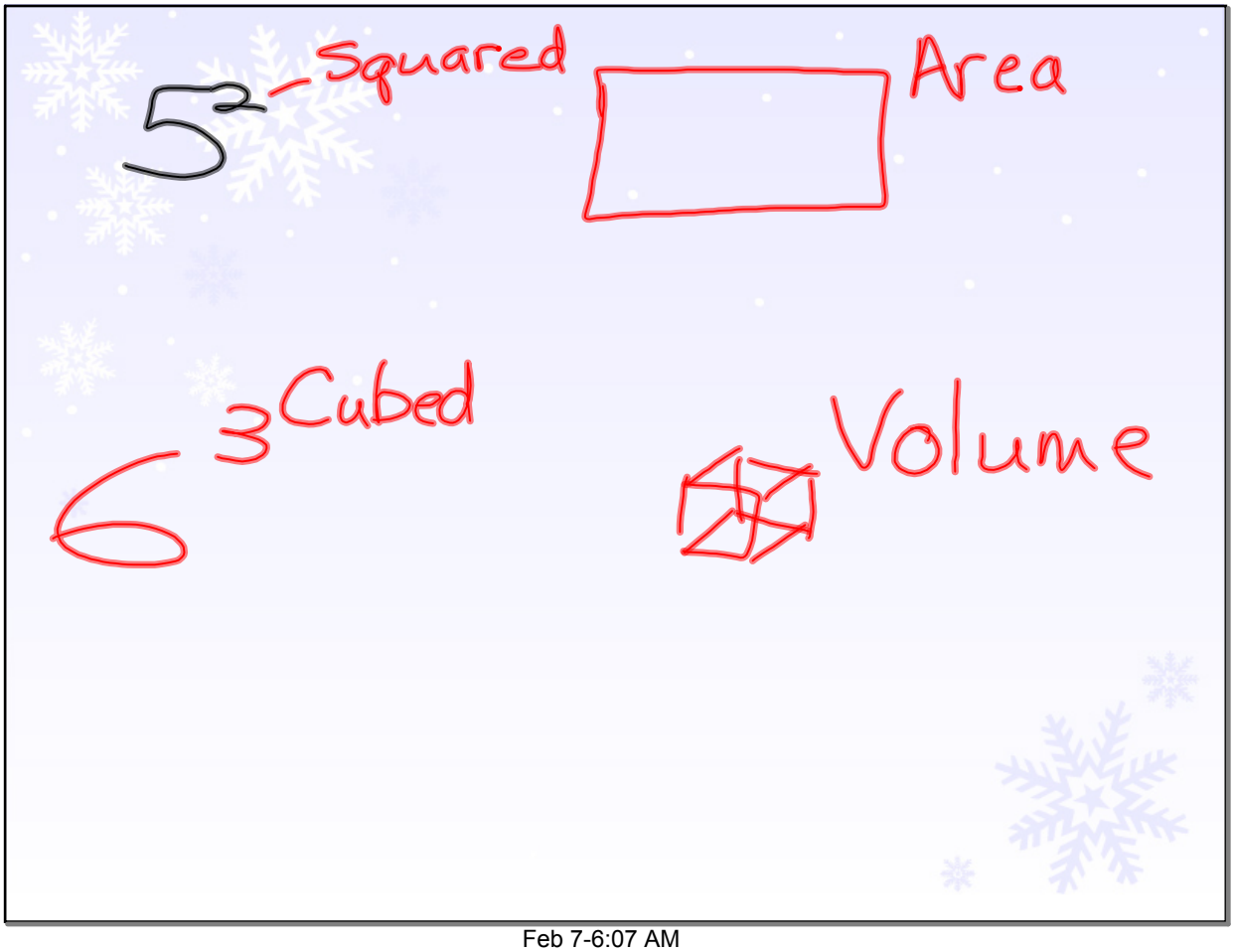

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Fill in the chart, supplying the missing expression.

1. Fill in the missing expression for each row. For whole number and decimal bases, use a calculator to find the standard form of the number. For fraction bases, leave your answer as a fraction.

Exponential Form	Written as a Multiplication Expression Having Repeated Factors	Standard Form
3^2	3×3	9
	$2 \times 2 \times 2 \times 2 \times 2 \times 2$	
4^5		
	$\frac{3}{4} \times \frac{3}{4}$	
	1.5×1.5	

2. Write "five cubed" in all three forms (exponential form, written as a series of products, standard form)

3. Write "fourteen and seven tenths squared" in all three forms.

4. One student thought two to the third power was equal to six. What mistake do you think they made and how would you help them fix their mistake?

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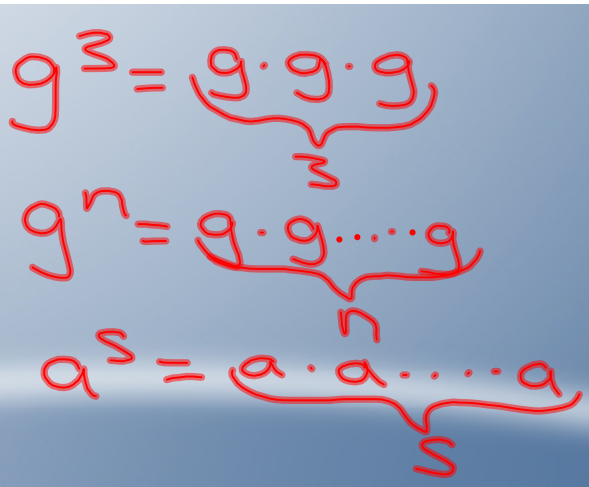
Lesson Summary

Exponential Notation for Whole Number Exponents: Let m be a non-zero whole number. For any number a, the expression a^m is the product of m factors of a, i.e.,

$$a^m = \underbrace{a \cdot a \cdot \cdots \cdot a}_{m \text{ times}}.$$

The number a is called the *base*, and m is called the *exponent* or *power* of a.

When m is 1, "the product of one factor of a" just means a, i.e., $a^1 = a$. Raising any non-zero number a to the power of 0 is defined to be 1, i.e., $a^0 = 1$ for all $a \neq 0$.



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Problem Set

1. Complete the table by filling in the blank cells. Use a calculator when needed.

Exponential Form	Written as a Series of Products	Standard Form
3^5	$3 \times 3 \times 3 \times 3 \times 3$	243
	$4 \times 4 \times 4$	
1.9^2		
$\left(\frac{1}{2}\right)^5$		

=Answer

2. Why do whole numbers raised to an exponent get greater while fractions raised to an exponent get smaller?

3. The powers of 2 that are in the range 2 through 1,000 are 2, 4, 8, 16, 32, 64, 128, 256, and 512. Find all the powers of 3 that are in the range 3 through 1,000.

4. Find all the powers of 4 in the range 4 through 1,000.

5. Write an equivalent expression for $n \times a$ using only addition.

6. Write an equivalent expression for w^b using only multiplication.
   a. Explain what w is in this new expression.
   b. Explain what b is in this new expression.

7. What are the advantages to using exponential notation?

8. What is the difference between $4x$ and x^4? Evaluate both of these expressions when $x = 2$.

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